```
$TITLE APPLICATION OF THE LOGIC-BASED MINLP ALGORITHM IN EXAMPLE #3
* THE FORMULATION IS DISJUNCTIVE
$OFFSYMXREF
$OFFSYMLIST
*  SELECT OPTIMAL PROCESS FROM WITHIN GIVEN SUPERSTRUCTURE.
*
SETS   I      PROCESS STREAMS                 / 1*25 /
       J      PROCESS UNITS                   / 1*8 /

PARAMETERS   CV(I)     VARIABLE COST COEFF FOR PROCESS UNITS - STREAMS
       /  3 = -10  ,  5 = -15  ,  9 = -40,   19 =  25  , 21 =  35  , 25 = -35
         17 = 80  , 14 =  15  , 10 =  15,    2 =  1   ,  4 =  1   , 18 = -65
         20 = -60  , 22 = -80  /;

 VARIABLES  PROF     PROFIT ;

 BINARY VARIABLES    Y(J)         ;
 POSITIVE VARIABLES  X(I) , CF(J);

 EQUATIONS
* EQUATIONS Independent of discrete choices
* ----------------------------------------------------------
 MASSBAL1, MASSBAL2, MASSBAL3, MASSBAL4, MASSBAL5, MASSBAL6, MASSBAL7, MASSBAL8
 SPECS1, SPECS2, SPECS3, SPECS4

* EQUATIONS allowing flow just IFF the unit EXISTS
* ---------------------------------------------------
 LOGICAL1, LOGICAL2, LOGICAL3, LOGICAL4, LOGICAL5, LOGICAL6, LOGICAL7, LOGICAL8


* DISJUNCTION'S CONSTRAINTS and EQUATIONS
* ----------------------------------------
 INOUT11, INOUT12, INOUT13, INOUT14  INPUT-OUTPUT RELATIONS FOR PROCESS UNIT 1
 INOUT21, INOUT22, INOUT23, INOUT24  INPUT-OUTPUT RELATIONS FOR PROCESS UNIT 2
 INOUT31, INOUT32, INOUT34           INPUT-OUTPUT RELATIONS FOR PROCESS UNIT 3
 INOUT41, INOUT42, INOUT43, INOUT44, INOUT45                FOR PROCESS UNIT 4
 INOUT51, INOUT52, INOUT53, INOUT54  INPUT-OUTPUT RELATIONS FOR PROCESS UNIT 5
 INOUT61, INOUT62, INOUT63, INOUT64  INPUT-OUTPUT RELATIONS FOR PROCESS UNIT 6
 INOUT71, INOUT72, INOUT73, INOUT74  INPUT-OUTPUT RELATIONS FOR PROCESS UNIT 7
 INOUT81, INOUT82, INOUT83, INOUT84, INOUT85, INOUT86       FOR PROCESS UNIT 8
 OBJETIVO          OBJECTIVE FUNCTION DEFINITION ;

* BOUNDS SECTION:
* ---------------
 X.UP('3')  =  2.0 ;
 X.UP('5')  =  2.0 ;
 X.UP('9')  =  2.0 ;
 X.UP('10') =  1.0 ;
 X.UP('14') =  1.0 ;
 X.UP('17') =  2.0 ;
 X.UP('19') =  2.0 ;
 X.UP('21') =  2.0 ;
 X.UP('25') =  3.0 ;

 OPTIONS LIMCOL = 0   ;
 OPTION LIMROW = 0    ;
 OPTION OPTCR = 0     ;
*DEFINITIONS of  EQUATIONS Independent of discrete choices
 MASSBAL1..   X('13')         =E=  X('19') + X('21')         ;
 MASSBAL2..   X('17')         =E=  X('9') + X('16') + X('25') ;
 MASSBAL3..   X('11')         =E=  X('12') + X('15')         ;
 MASSBAL4..   X('3') + X('5') =E=  X('6') + X('11')          ;
 MASSBAL5..   X('6')          =E=  X('7') + X('8')           ;
 MASSBAL6..   X('23')         =E=  X('20') + X('22')         ;
 MASSBAL7..   X('23')         =E=  X('14') + X('24')         ;
 MASSBAL8..   X('1')          =E=  X('2') + X('4')           ;
 SPECS1..     X('10')  =L=  0.8 * X('17')                    ;
 SPECS2..     X('10')  =G=  0.4 * X('17')                    ;
 SPECS3..     X('12')  =L=  5.0 * X('14')                    ;
 SPECS4..     X('12')  =G=  2.0 * X('14')                    ;

* DEFINITION of EQUATIONS allowing flow just IFF the unit EXISTS
 LOGICAL1..   X('2') + X('3')   =L=  10. * Y('1')          ;
 LOGICAL2..   X('4') + X('5')   =L=  10. * Y('2')          ;
 LOGICAL3..   X('9')   =L=  10. * Y('3')                   ;
 LOGICAL4..   X('12') + X('14')  =L=  10. * Y('4')         ;
 LOGICAL5..   X('15')  =L=  10. * Y('5')                   ;
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 LOGICAL6..   X('19')  =L=  10. * Y('6')                   ;
 LOGICAL7..   X('21')  =L=  10. * Y('7')                   ;
 LOGICAL8..   X('10') + X('17')  =L=  10. * Y('8')         ;

*DEFINITIONS of DISJUNCTION's EQUATIONS
 INOUT11..   EXP(X('3')) -1. =E= X('2')            ;
 INOUT14..   CF('1') =E= 5                         ;
 INOUT12..   X('2') =E= 0                          ;
 INOUT13..   X('3') =E= 0                          ;
 INOUT21..   EXP(X('5')/1.2) -1. =E= X('4')        ;
 INOUT24..   CF('2') =E= 8                         ;
 INOUT22..   X('4') =E= 0                          ;
 INOUT23..   X('5') =E= 0                          ;
 INOUT31..   1.5 * X('9') + X('10') =E= X('8')     ;
 INOUT34..   CF('3') =E= 6                         ;
 INOUT32..   X('9') =E= 0                          ;
 INOUT41..   1.25 * (X('12')+X('14')) =E= X('13')  ;
 INOUT45..   CF('4') =E= 10                        ;
 INOUT42..   X('12') =E= 0                         ;
 INOUT43..   X('13') =E= 0                         ;
 INOUT44..   X('14') =E= 0                         ;
 INOUT51..   X('15') =E= 2. * X('16')              ;
 INOUT54..   CF('5') =E= 6                         ;
 INOUT52..   X('15') =E= 0                         ;
 INOUT53..   X('16') =E= 0                         ;
 INOUT61..   EXP(X('20')/1.5) -1. =E= X('19')      ;
 INOUT64..   CF('6') =E= 7                         ;
 INOUT62..   X('19') =E= 0                         ;
 INOUT63..   X('20') =E= 0                         ;
 INOUT71..   EXP(X('22')) -1. =E= X('21')          ;
 INOUT74..   CF('7') =E= 4                         ;
 INOUT72..   X('21') =E= 0                         ;
 INOUT73..   X('22') =E= 0                         ;
 INOUT81..   EXP(X('18')) -1. =E= X('10') + X('17');
 INOUT86..   CF('8') =E= 5                         ;
 INOUT82..   X('10') =E= 0                         ;
 INOUT83..   X('17') =E= 0                         ;
 INOUT84..   X('18') =E= 0                         ;
 INOUT85..   X('25') =E= 0                         ;

 OBJETIVO  .. PROF  =E= SUM(J,CF(J)) + SUM(I , X(I)*CV(I)) + 122 ;

* BEGIN DECLARATIONS AND DEFINITIONS OF DISJUNCTIONS (LOGMIP Section)
$ONECHO > "%lm.info%"
disjunction  d1, d2, d3, d4, d5, d6, d7, d8;

       d1 is  if Y('1') then
                INOUT11;
                INOUT14;
               else
                INOUT12;
                INOUT13;
               endif;

       d2 is  if Y('2') then
                INOUT21;
                INOUT24;
               else
                INOUT22;
                INOUT23;
               endif;

       d3 is if Y('3') then
                INOUT31;
                INOUT34;
               else
                INOUT32;
               endif;

       d4 is if Y('4') then
                INOUT41;
                INOUT45;
               else
                INOUT42;
                INOUT43;
                INOUT44;
               endif;
```

```
        d5 is if Y('5') then
                 INOUT51;
                 INOUT54;
                else
                 INOUT52;
                 INOUT53;
                endif;

        d6 is if Y('6') then
                 INOUT61;
                 INOUT64;
                else
                 INOUT62;
                 INOUT63;
                endif;

        d7 is if Y('7') then
                 INOUT71;
                 INOUT74;
                else
                 INOUT72;
                 INOUT73;
                endif;

        d8 is  if Y('8') then
                 INOUT81;
                 INOUT86;
                else
                 INOUT82;
                 INOUT83;
                 INOUT84;
                 INOUT85;
                endif;
```

```
         atmost(Y('1'), Y('2'));
         atmost(Y('4'), Y('5'));
         atmost(Y('6'), Y('7'));
```

Logic constraints expressed by special sentences

```
         Y('1') -> Y('3') or Y('4') or Y('5');
         Y('2') -> Y('3') or Y('4') or Y('5');
         Y('3') -> Y('8');
         Y('3') -> Y('1') or Y('2');
         Y('4') -> Y('1') or Y('2');
         Y('4') -> Y('6') or Y('7');
         Y('5') -> Y('1') or Y('2');
         Y('5') -> Y('8');
         Y('6') -> Y('4');
         Y('7') -> Y('4');
```

Constraints in the form of logic propositions

```
         INIT TRUE Y('2'), Y('3'), Y('4'), Y('6'),
         INIT TRUE Y('1'), Y('3'), Y('4'), Y('7'),
         INIT FALSE Y('2'), Y('4'), Y('6'), Y('7');
```

This is a special section for non-linear problems. This is an initialization section needed by the LOGIC BASED OA algorithm, see Turkay and Grossmann (1996a).

```
$OFFECHO
* end logmip section

 OPTION MINLP=LMLBOA;

 MODEL EXAMPLE3  / ALL / ;
 SOLVE EXAMPLE3 USING MINLP MINIMIZING PROF   ;
```

LMLBOA is the solver for non-linear problems, which applies the LOGIC_BASED OA algorithm. You need a NLP and a MIP solver installed together with GAMS to solve this problem.